Exam. Code : 103202
Subject Code : 1026

B.A./B.Sc. 2nd Semester

MATHEMATICS

Paper—II (Calculus)

Time Allowed—2 Hours] [Maximum Marks—50

Note :— There are ***eight*** questions of equal marks. Candidates are required to attempt any ***four*** questions.

1. (a) Discuss the continuity of the function :

$$f(x, y) = \begin{cases} xy\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right); & (x, y) \neq (0,0) \\ 0 & ; (x, y) = (0,0) \end{cases}$$

at the point (0, 0).

(b) If $f(x, y) = \log(x^2 + y^2) + \tan^{-1}\dfrac{y}{x}$, then prove $fxx + fyy = 0$.

(c) Let $f(x, y) = \begin{cases} xy\left(\dfrac{x - y}{x + y}\right); & (x, y) \neq (0,0) \\ 0 & ; (x, y) = (0,0) \end{cases}$

show that $fxy\ (0, 0) \neq fyx(0, 0)$.

2. (a) Discuss the differentiability of $f(x, y) = \sin x + \sin y$; $(x, y) \in R^2$.

(b) Show that for the function :

$$f(x, y) = \begin{cases} \left(\dfrac{x^2 - y^2}{x^2 + y^2}\right); & (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

conditions of Young's Theorem are not satisfied, even though $fxy(0, 0) = fyx(0, 0)$.

(c) Show that the equation $x^2 + xy + y^2 = 7$ defines y as a function of x in the neighbourhood of the point (2, 1). Find derivatives of this function at $x = 2$.

3. (a) If $z = \sin^{-1}\left(\dfrac{x^2 + y^2}{x + y}\right)$, prove that $x\dfrac{dz}{dx} + y\dfrac{dz}{dy} = \tan z$.

(b) Obtain Taylor expression for $f(x, y) = x^2 + xy + y^2$ about (1, 2) upto third term.

(c) If $y_1 = \cos x_1$, $y_2 = \sin x_1 \cos x_2$, $y_3 = \sin x_1, \sin x_2, \cos x_3$ .... $y_n = \sin x_1, \sin x_2,$ .... $\sin x_{n-1} \cos x_n$, then find $\dfrac{\partial(y_1 y_2 \ldots\ldots, y_n)}{\partial(x_1 x_2 \ldots\ldots, x_n)}$.

4. (a) Find the envelope of the family of circles $x^2 + y^2 - 2ax \cos\theta - 2ay \sin\theta = c^2$ where θ is a parameter.

(b) Find all the points of maxima and minima of the function $f(x, y) = x^3 + y^3 - 63(x, y) + 12xy$. Also discuss the saddle points (if any) of the function.

(c) Find the evolute of the parabola $y^2 = 4ax$ regarding it as an envelope of its normal.

5. (a) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$.

(b) Change the order of integration and hence evaluate the integral $\displaystyle\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$

6. (a) Evaluate $\displaystyle\iiint \frac{dx\,dy\,dz}{\sqrt{1 - x^2 - y^2 - z^2}}$ over the positive octant of sphere $x^2 + y^2 + z^2 = 1$.

(b) Evaluate $\displaystyle\iiint z(x^2 + y^2)dx\,dy\,dz$ over the region $x^2 + y^2 \leq 1$, $2 \leq z \leq 3$.

7. (a) Find the area of the region bounded by $y = 2 - x^2$ and $y = x$.

(b) Find the area enclosed by $y^2 = x^3$ and $y = x$.

8. (a) Find the volume of the right circular cone with basie radius r and height h.

(b) Find the volume bounded by the surfaces $x^2 + y^2 = a^2$ and $\dfrac{x^2}{r} + \dfrac{y^2}{s} = 2z$ where $r, s > 0$.